COSMICAL AND MOLECULAR HARMONIES, NO. II.

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I. Harmonic Indications of Intra-Mercurial Planets.

The modification of the planetary harmonic series, by the inertia of large planetary masses, is perhaps, no less interesting than the primitive series themselves.

If we take the Neptunian radius vector as our unit, the linear centre of oscillation as our prime determinant, and the oscillatory ratio, 3, as our harmonic difference, we obtain the harmonic series,

$$\Psi \quad \tfrac{2}{3} \quad \tfrac{2}{6} \quad \tfrac{2}{9} \quad \tfrac{2}{12}, \text{ \&c.}$$

The first term of this series represents the secular mean aphelion of Uranus ; the second, Saturn's mean aphelion ; the third, Saturn's aphelion centre of linear oscillation, as well as the mean centre of gravity of the planetary system ; the fourth, Jupiter's mean perihelion. Jupiter's mean perihelion is at the octave node of Saturn's mean aphelion, and their joint harmonic importance has been amply illustrated in my previous papers.

The regularity of this series is interrupted by the influence of the great masses of Saturn and Jupiter, and although inferior planetary positions may be approximately represented by subsequent terms, they are found only at every eighth term of the Neptunian, or at every term of the simple Jupiter series.

$$♃ \quad \tfrac{1}{1} \quad \tfrac{1}{3} \quad \tfrac{1}{5} \quad \tfrac{1}{7} \quad \tfrac{1}{9} \quad \tfrac{1}{11} \quad \tfrac{1}{13} \quad \tfrac{1}{15}$$

or,

$$\Psi \quad \tfrac{2}{12} \quad \tfrac{2}{36} \quad \tfrac{2}{60} \quad \tfrac{2}{84} \quad \tfrac{2}{108} \quad \tfrac{2}{132} \quad \tfrac{2}{156} \quad \tfrac{2}{180}$$

These terms represent, in regular succession, Jupiter's mean perihelion ; the mean aphelion of Mars ; the mean distances of Earth and Venus ; the mean octave node of Venus and Mercury ; Mercury's mean aphelion ; Mercury's mean distance ; and the Earth's reverse centre of linear oscillation.

The complete continuity of this series, like that of the foregoing, is broken by the combined disturbance of Earth and Venus, but its thirteenth term ($\tfrac{1}{25}$ ♃ or $\tfrac{2}{300}$ Ψ) approximates quite nearly to Kirkwood's estimated mean distance of Vulcan (.209 $\oplus$). A still closer accordance is afforded by the following harmonic series, which assumes Earth's mean distance as the unit.

$$\oplus \quad \frac{7}{2} \quad \frac{7}{10} \quad \frac{7}{18} \quad \frac{7}{26} \quad \frac{7}{34}$$

The first term of this series represents Jupiter's linear centre of oscillation, at which the planetary masses would balance the Sun, with a centre of gravity at the Sun's surface, which is the source of its radiant undulation ; the second, the mean distance of Venus ; the third, Mercury's mean distance ; the fourth, a possible unknown planet, planetoid group, or other seat of solar and planetary perturbation, with a period of 53.54 $\pm$ days ; the fifth, a planet with a period of 34.25 days, Kirkwood's estimated period being 34.92 days.

It is especially noticeable that in these three harmonic series, embracing sixteen terms in all, only one appears to be destitute of an obvious planetary representative. The lack of actual known planets at some of the principal centres of linear oscillation, may, perhaps, be accounted for by the action of masses, which, though not sufficient to introduce new series, may have concurred in breaking up the tendency to nodal aggregation. It is possible that future observations may show a somewhat analogous action between Mercury and Vulcan. I am, however, inclined to believe that improvements in astronomical instruments will, at some day, enable careful observers to detect some form of actual material concentration, at $\frac{2}{9}$ ♆, $\frac{1}{9}$ ♃, $\frac{2}{3}$ ♃, and $\frac{7}{26}$ $\oplus$.

These harmonic pointings to intra-Mercurial planets are corroborated by the π series of planetary pairs, which was given in my communication of the 4th *ultimo*. The division of Mercury's perihelion distance by π brings us to a point considerably within the limits of the solar retardation, which, if the æther is material, would interfere with any permanent orbital revolution, bringing any planetary masses more or less rapidly to the Sun. There is, however, a certain portion of the region in which planetary stability would be possible, and analogy would lead us to look for a pair of mutually balancing planets, wherever the conditions are favorable for one. The closeness of the harmonic accordance in the position which Kirkwood has assigned to Vulcan, seems to me a strong confirmation of his views, and an equally strong indication of the importance of making special observations in the portion of the Zodiacal belt which is embraced between the orbits of Vulcan and Mercury. If Wolf's supposed Sun-spot period of twenty-seven days* is a sidereal period, it might be readily explained by the perturbations and transits of a planetoid or meteoric group, at a distance which would complete the terrestrial harmonic series. If the proper balancing of orbital undulations requires any extra Neptunian planets, we may, perhaps, have reason to look for some analogy between them and the intra-Mercurial pair, which may complete the symmetry between the outer and inner planetary limits to which I have invited attention in my modification of Bode's Law.

* Cited by Kirkwood, *ante*, xi. 97.

The following table exhibits the theoretical and observed values for each of the forgoing series :

HARMONIC NODES IN THE SOLAR SYSTEM.

♆ Series.	♃ Series.	⊕ Series.	Theoretical.	Observed.	
$\frac{2}{3}$			20.025	20.043	♅ a^* = ♆ c. o.
$\frac{2}{6}$			10.012	10.000	♄ a
$\frac{2}{9}$			6.675	6.667	♄ c. o. = $\frac{1}{2}$ ♅ c. o.
$\frac{2}{12}$	$\frac{1}{1}$		5.006	4.978	♃ p^*
$\frac{2}{36}$	$\frac{1}{3}$		1.669	1.644	♂ a
$\frac{2}{60}$	$\frac{1}{5}$		1.001	1.000	⊕
$\frac{2}{84}$	$\frac{1}{7}$		.715	.723	♀
$\frac{2}{108}$	$\frac{1}{9}$		.556	.555	♀ ☿
$\frac{2}{132}$	$\frac{1}{11}$		.455	.455	☿ a
$\frac{2}{156}$	$\frac{1}{13}$		.385	.387	☿
$\frac{2}{180}$	$\frac{1}{15}$		.334	.333	⊕ r. c. o.
	$\frac{7}{10}$	$\frac{7}{2}$	3.500	3.469	♃ c. o.
	$\frac{7}{50}$	$\frac{7}{10}$	.700	.723	♀
	$\frac{7}{90}$	$\frac{7}{18}$	.389	·387	☿
	$\frac{7}{130}$	$\frac{7}{26}$	.269	.269?	(?)
	$\frac{7}{170}$	$\frac{7}{34}$	.206	.209	Vulcan.
Sum......................			51.897	51.842	

II. CORRELATIONS OF PLANETARY MASS.

The Sun's surface, which is the seat of the mutual action and reaction of its attractive and radiating forces, is also the fulcrum of a lever which would balance the solar and planetary masses if the latter were placed near the outer asteroidal limit, or near the linear centre of oscillation of Jupiter's radius vector. The outer end of the lever is also at a distance (3.513 ⊕), at which the velocity of revolution is equivalent to the velocity which would give Jupiter, at its perihelion, a parabolic orbit. These harmonies seem to point to harmonies of mass, which, by means of commensurate momenta or moments of inertia, may help to sustain the equipoise of the system.

I have already pointed out, in my note on "Oscillatory Forces in the Solar System" (*ante, p.* 140), some striking, and, I believe, entirely novel evidences of such harmony, which seem to be attributable to the same play of elastic forces which has assigned to each planet its appropriate position. There are still further and somewhat analogous relations between the centres of gravity of the two principal pairs of planets, as well as between solar and important planetary positions, which I submit as additional confirmation of the influence of æthereal vibration.

My hypothetical arrangement of æthereal particles (*ante,* xii, 408),

**a* — Aphelion; *p* — Perihelion.

suggests a primary planetary ellipse in which the transverse is to the conjugate axis in the proportion of 2 to $\sqrt{3}$. In such an ellipse the two foci and either extremity of the transverse axis represent the extremes and centre of oscillation of a linear pendulum; parallels to the lines joining the foci to either end of the conjugate axis, trisect the quadrant and semicircle; the virtual velocity of oscillation at the end of the supposed rigid transverse axis, is to the velocity at the end of the conjugate axis, as the velocity of infinite fall to the extremity of the transverse axis is to the velocity of revolution at its linear centre of oscillation.

Let A and D represent the ends of the transverse axis, B and C the foci of the hypothetical ellipse. Let *a* denote planetary aphelion; *p*, perihelion; *m*, centre of gravity, at secular mean conjunction, of Neptune and Uranus; *n*, mean conjunctive centre of gravity of Jupiter and Saturn at Jupiter's mean aphelion and Saturn's mean perihelion; n^1, centre of gravity at Jupiter's mean perihelion and Saturn's mean aphelion. The four principal planetary masses are so proportioned to their distances, and their distances are so proportioned to those of Mars and the Earth, as to give the following arrangements in the typical ellipse.

	A	B	Centre.	C	D
1	$m\ p$	☉	.	.	$m\ a$
2	☉	♃ p	.	.	♅ a
3	$n\ p$	☉	.	.	$n\ a$
4	$n_1 a$	♃ p	.	.	$n_1 p$
5	☉	2 ♂ p	.	$m\ p$	.
6	☉	2 ♂	.	♄ p	.
7	☉	2 ♂ a	.	♄ a	.
8	♂	⊕	.	☉	.

It is well to observe that the relation of Mars's radius vector to octave nodes of the asteroidal belt is indicated by these configurations, as well as by the π series in my communication of April 4th (π^{-2} ♆ = 2 ♂).

If we accept Newton's hypothesis, that the æther is a material medium, by means of which the mutual gravitating action of masses is exerted, the modulus of light may be determined by the same laws as the modulus of elasticity in air, steel, or other terrestrial elastic bodies, and its determination gives significance to the following relations:

1. The half-modulus, is to Jupiter's distance from the Sun's surface, as the Earth's distance from the Sun's surface, is to solar radius (or the distance of the Sun's centre of gravity from its surface).

2. The quotient of the solar mass by the aggregate planetary mass is to the quotient of the mean aphelion by the mean perihelion radius vector of the Solar-Jovian centre of gravity, as the velocity of light is to the velocity of planetary revolution at the mean perihelion Solar-Jovian centre of gravity.

3. The aggregate mass of the Jovian System (♃, ♄, ♆, ♅), is to that of the Telluric System (⊕, ♀, ♂, ☿), as Earth's radius vector, is to Sun's radius.

4. The aggregate mass of the principal planetary pair in the extra-asteroidal belt (♃, ♄), is to the mass of the principal planetary pair in the intra-asteroidal belt (⊕, ♀), as the Earth's distance from the Sun, is to solar radius.

5. The aggregate mass of the exterior planetary pair (♆, ♅), is to the mass of Saturn, as Saturn's mean aphelion distance, is to Neptune's mean distance from the Sun.

6. The aggregate mass of the smaller planetary pair in the terrestrial belt (♂, ☿), is to the mass of the larger pair (⊕, ♀), as the force of solar gravitation in any planetary orbit, is to the force at the planet's reverse centre of linear oscillation (1 : 9).

7. The aggregate mass of the inner planetary pair of the terrestrial belt (♀, ☿), is to the mass of the outer pair (⊕, ♂), as twice the square of the radius of spherical gyration, is to the square of the equatorial radius. The uncertainty as to the mass of Mercury, renders the last two ratios more doubtful than the three preceding. In order to make them equally satisfactory, it may be necessary to take account of intra-Mercurial planets or planetoids.

8. The mean annual motion of Neptune's perihelion, is one-sixth of that of Uranus; that of Uranus being one-sixth of that of Saturn. The mean perihelion distance of Mars, is one-sixth of the distance of mean centre of gravity of Neptune and Uranus at their opposition. Its mean distance is one-sixth of Saturn's mean perihelion distance; its mean aphelion distance, one-sixth of Saturn's mean distance.

9. Terrestrial superficial gravity, is to solar superficial gravity, as Sun's radius, is to one-third of the mean distance of Mercury from the Sun's surface.

10. There are numerous relations between the varying positions of the center of gravity of Saturn and Jupiter, which seem to corroborate the nebular hypothesis and to encourage careful investigation.

11. The aggregate mass of Jupiter and Mars, is to the aggregate mass of Saturn and Earth, as the quotient of Saturn's aphelion by Earth's mean distance, is to the quotient of Jupiter's perihelion by Mars's aphelion.

12. The aggregate mass of Earth and Mercury, is to the aggregate mass of Venus and Mars, as the quotient of Mars's mean aphelion by the radius of spherical gyration in Earth's orbital æthereal sphere, is to the quotient of Earth's perihelion distance by Mercury's aphelion distance. This proportion, like the fourth and fifth, is affected by the uncertainty of Mercury's mass.

13. The Sun's radius is a mean proportional between its centre of gyration and the conjunctive centre of gravity of Sun, Jupiter's perihelion, and Saturn's perihelion.

14. The quotient of Sun's mass by Jupiter's mass, is to the fourth power of the quotient of the time of planetary revolution by the time of fall to the orbital center, as the mean perihelion distance of the center of gravity of Sun and Jupiter, is to Sun's radius.

15. The solar mass, is to the aggregate planetary mass, as the square of the gravitating force at the Sun's surface is to the square of the gravitating force at the Earth surface.

16. The solar mass, is to the aggregate planetary mass, as Sun's radius is to $(\frac{1}{9})^3$ of the secular mean radius vector of the perihelion center of gravity of Sun and Jupiter.

17. The solar mass, is to the aggregate planetary mass, as the gravitating force at the Sun's surface, is to the gravitating force at the linear centre of oscillation between Mercury and the Sun's surface.

18. Planetary velocity at the mean perihelion center of gravity of Sun and Jupiter is equal to $(\frac{1}{9})^3$ of the velocity of light.

19. Jupiter's mass, is to Saturn's mass, as $\frac{1}{3}$ of Saturn's mean aphelion distance, is to Earth's mean distance from the Sun.

20. Saturn's mass, is to Earth's mass, as the quotient of Saturn's mean aphelion distance by Earth's radius, is to the quotient of Earth's mean distance by Saturn's radius.

21. Saturn's mass, is to Neptune's mass, as the quotient of Jupiter's mean aphelion distance by the distance of the mean perihelion center of gravity of Sun and Jupiter, is to the quotient of Earth's mean distance by Sun's radius.

22. The mass of Uranus, is to Earth's mass, as the vector-radial quotient of Uranus's center of oscillation by Sun's radius, is to the quotient of Earth's mean distance by mean perihelion centre of gravity of Sun and Jupiter.

23. The mass of Uranus, is to Neptune's mass, as $\pi : 4$.

24. The mass of Venus, is to Earth's mass, as $\pi : 4$.

25. The mass of Jupiter, is to the mass of Neptune, as a mean proportional between Saturn's mean aphelion distance and Neptune's mean perihelion, is to the Earth's mean perihelion distance.

26. The mass of Jupiter, is to Earth's mass, as the product of Saturn's mean aphelion distance by Neptune's mean distance, is to the square of Earth's mean distance.

27. Neptune's mass, is to Earth's mass, as the continued vector-radial product of Sun and Jupiter's mean perihelion centre of gravity, Mars's mean aphelion, and Saturn's mean aphelion, is to the product of Sun's radius by the square of Earth's mean distance.

COMPARATIVE TABLE OF THEORETICAL AND OBSERVED VALUES.

		Theoretical.	Observed.
Mean Inertial Moment of Jupiter		Equal.	258,200
" " Saturn			259,627
" " Uranus $\times \pi$		"	482,378
" " Neptune			480,439
$\frac{1}{2}$ Light Modulus ÷ ♃,	(1)	213.36	213.71
Velocity of Light,	(2)	184,200	183,450
♃ System ÷ ⊕ System,	(3)	214.86	214.88
(♃, ♄) ÷ (⊕, ♀),	(4)	214.86	217.62

		Theoretical.	Observed.
(♆, ♅) ÷ ♄,	(5)..........	3.004	3.000
♂, Mean Perihelion,	(8)..........	1.4048	1.4032
♂, Mean,	"	1.5129	1.5237
♂, Mean Aphelion,	"	1.6667	1.6442
☉ *g* ÷ ⊕ *g*	(9)...........	27.39	27.29
(♃, ♂) ÷ ♄, ⊕),	(11)..........	3.3027	3.3091
(☉, ♃, ♄) Center of Gravity,	(13)..........	1.5812	1.5741
☉ ÷ ♃,	(14)..........	1044.25	1048.88
☉ ÷ Planets,	(15)..........	744.86	747.15
" "	(16)..........	743.41	747.15
" "	(17)..........	750.27	747.15
Velocity of Light,	(18)..........	183,570	183,450
♃ ÷ ♄,	(19)..........	3.3333	3.3448
♄ ÷ ⊕,	(20)..........	90.621	89.408
♄ ÷ ♆,	(21)..........	5.322	5.347
♅ ÷ ⊕,	(22)..........	13.041	13.083
♅. ÷ ♆,	(23)..........	.7854	.7825
♀ ÷ ⊕,	(24)..........	.7854	.7845
♃ ÷ ♆,	(25)..........	17.848	17.886
♃ ÷ ⊕,	(26)..........	300.37	299.05
♆ ÷ ⊕,	(27)..........	16.767	16.720

The observed values in this table are computed from the data given by Norton and Stockwell.

III. Harmonies of Cosmical Rotation.

The numerous indications of arithmetical, harmonic, and geometrical progression which I have pointed out, are such as might have been anticipated, if we regard the æther as a material medium of extreme elasticity, by means of which all forms of material force are transmitted.

Chladni's experiments gave abundant evidence of the tendency of sonant vibrations to produce nodal and symmetrical aggregations of material particles ; the drinking-glass experiments show that the tendency is sometimes sufficient to overcome cohesive or other attractions which oppose the symmetry ; Mossotti's hypothesis, that the radius of each particular material nucleus may be almost infinitesimal in comparison with the radius of the æthereal atmosphere by which it is surrounded, makes it comparatively easy to believe that even suns and planets may be influenced as to position, volume, mass, and motion, by harmonious vibrations which are transmitted from particle to particle until they pervade and control the whole mass.

Rotation, as well as revolution, is one form of oscillation, and therefore subject to the same general laws as the swing of pendulums, the undulation of liquids, the sonant and luminous tremors of elastic solids, fluids, and æthers. Some evidences of its being subject to rhythmical influences may be found in the approximate agreement between the lengths of days both in the Jovian and in the Telluric planetary belt. Other and more

satisfactory evidences are deducible from such comparisons of uniform and variable motions as are naturally suggested by the foregoing investigations.

Among the correlations in the preceding article, the following seem peculiarly significant:

1. The close coincidence between the vectorial ratio of Jupiter to the half-modulus of light, and that of Sun's radius to Earth; Jupiter and Earth being the controlling masses of their respective planetary belts.

2. The ratio which connects the velocity of light, the proportionate masses of Sun and Planets, the velocity of planetary revolution about the Sun, the variations of position in the center of gravity of the two principal masses of the System (☉, ♃), and the consequent orbital eccentricity of Jupiter.

3. The comparative commensurability of the Jovian and Telluric Systems, Sun's radius and Earth's radius vector.

13. The mean proportionality of Sun's radius between the radius of its centre of gyration and the radius of the centre of gravity of the three principal masses of the System (☉, ♃, ♄).

n, n^1. The ratios which fix the elliptical orbits of the relative mean aphelia and perihelia of Jupiter and Saturn and the eccentricity of Mars, the planet which links the Telluric to the asteroidal belt.

I have already invited attention to the approximate mean proportionality between the distances of Mercury and Neptune from the Sun's surface, and to the resemblances between the gamuts of sound and light. If we regard the condensation of planets and the gaseous elasticity of their envelopes as both resultants of æthereal elasticity, we may naturally look to logarithmic curves of the second order for some interesting comparisons.

If we divide the planetary octave into twenty-four ($= 3 \times 2^3$) quarter tones and consider Jupiter's mean perihelion as occupying the logarithmic centre of oscillation ($2^{\frac{2}{3}}$ *log.* ☿), we obtain the geometrical series of logarithms in column *ld* of the following table, the *ratios* of the logarithms being represented by the gamut ratios $(2)^{\frac{n}{24}}$. For comparison, I have also given the logarithms of actual distance, ld^1; the theoretical and actual distances, d, d^1; and the proportionate wave-lengths of the eight principal Fraunhofer lines *Fr.* Planetary perihelion, mean, and aphelion distances are represented, respectively by p, m, a, in accordance with Stockwell's estimates of the secular mean values.

LOGARITHMIC PLANETARY GAMUT.

	Fr.	$(2)^{\frac{n}{24}}$	*ld*	ld^1	d	d^1
☿ m	H 1.000	1.000	1.908294	1.919977	80.96	83.17
		1.029				
		1.059				
	G 1.095	1.091				
		1.122				

	Fr.	$(2)^{\frac{n}{24}}$	ld	ld^1	d	d^1
♀ a		1.155	2.204751	2.206566	160.23	160.90
		1.189				
⊙ m	F 1.235	1.224	2.335855	2.332155	216.70	214.86
		1.260				
		1.297				
♂ a	E 1.339	1.335	2.547265	2.548099	352.59	353.26
		1.374				
		1.414				
		1.456				
	D 1.498	1.498				
		1.542				
♃ p		1.587	3.029231	3.029231	1069.62	1069.62
		1.634				
	C 1.668	1.682				
♄ m	B 1.746	1.731	3.303395	3.311651	2010.92	2049.51
		1.782				
		1.834				
♅ p		1.888	3.602383	3.595130	4002.97	3936.68
	A 1.934	1.943				
♆ a		2.000	3.816597	3.814149	6555.36	6518.52

This hypothetical duplication of elastic influence, substitutes for the variable influence of gravity, an influence which is supposed to be uniform, in all places and at all distances. In order to find whether gravity can be represented by the resultant of such an influence, let

g = force of gravity at d.

d = perispheral distance of planet or satellite in parabolic orbit.

v_0 = " velocity " " " "

v_1 = velocity of rotation at spheroidal centre of gyration.

v_2 = " primary undulation which originates rotation and revolution.

t_0 = time of undulating action required to produce v_0.

d_0 = distance of virtual gravitating fall or of actual undulatory progression in t_0.

Then, $g \propto \frac{1}{d^2} \propto v_0{}^4 \propto t_0{}^4 \propto d_0{}^2$.

If an attracting spheroid were built up by aggregation of matter, urged simultaneously from all directions towards a resisting nucleus, the limit of velocity, v_0, would be retarded by internal work as soon as the particles began to come into mutual collision. This retardation would lead to centripetal condensation, with a

$$v \propto g \propto d.$$

By the law of equality of areas, if the sphere be imagined to be homogeneously expanded to a radius d, the mean velocity, or v_1, should vary as $\frac{1}{d}$; v_0, as $\left(\frac{1}{d}\right)^{\frac{1}{2}}$; and, v_2 being constant, $v_0 = \sqrt{v_1 v_2}$.

The values v_0 and v_1 are known, at least approximately, for Earth, Moon,

Sun, Mercury, Venus, Mars, Jupiter, Saturn; we have, therefore, the means of approximating to the velocity of primary undulation (v_2) with which each of those orbs is accordant, and perhaps, of lending indirect confirmation to the estimates of v_0 and v_1. The values in the following table are given in miles per hour; the computed values are marked C, the observed, O; the column of equivalents gives the velocities which are expressed numerically in the preceding column (v_2 O).

VELOCITY OF UNDULATIONS ACCORDANT WITH ROTATION.

	Log. v_0	Log. v_1	v_2 C.	v_2 O.	Equivalents.
⊕	4.397669	2.818189	948,743	946,550	plan v at c. g. ☉, ♃ p.
☽	3.722803	.816385	4,251,470	4,249,480	v light × ☉r ÷ ♀ r. vec.
☉	6.130913	3.446620	653,440,000	660,434,400	v light.
☿	4.121473	2.387331	717,158	716,900	plan v at $\frac{16}{9}$ ☉ r.
♁	4.355607	2.807660	800,856	801,860	" at 1.421 ☉ r.
♂	4.011997	2.598377	266,451	267,290	" at 9 × " "
♃	5.112849	4.248874	948,350	946,550	" at c. g. ☉ ♃ p.
♄	4.894799	4.133748	452.742	452,400	" at π × 1.421 ☉ r.

The most satisfactory of the accordances is undoubtedly the one identifying the velocity, of the primary undulation which is in simultaneous accordance both with the planetary velocities and with the velocity of solar rotation, with the velocity of light. This furnishes, as it seems to me, a crucial test of Newton's hypothesis that gravitating action is transmitted through the medium of the same æther to which Huyghens attributed the undulations of light. Challis's Hydrodynamic Researches (P. Mag., 1862, sqq.), and Norton's Investigations in Molecular Physics, (Am. Jour. of Sci., 1864, sqq,) have shown that the phenomena, both of attraction and of repulsion, can be produced by elastic undulation, and if the undulating velocity is the same in the two great sources of cosmical and molecular phenomena, solar radiation and attraction, what more can we ask for their complete identification, as opposite phases of a single primordial activity?

There is, however, still a discrepancy of about one per cent. between the computed and observed values of our supposed equivalents. This difference is exceedingly slight, it is true, and it could be readily overcome by adopting values, for the period of solar rotation and the solar radius, which are quite as probable as those I have employed. But such a forced agreement would render the result less, rather than more satisfactory. It is well known that there is a degree of uncertainty in each of the elements of the equation, greater than the one indicated by the result. This uncertainty must therefore remain, until it is reduced by more accurate observations, more careful study of modifying influences, and more complete comparison of such investigations, in various fields of research, as may help to illustrate the nature and degree of æthereal elasticity and the laws of its action.

In the case of the Moon, we have a virtual particle resting on the sur-

face of a rotating æthereal sphere, and therefore synchronous in its periods of rotation and revolution. The mean velocity of the spheral synodic rotation. ($2\pi \times 238{,}800 \times \sqrt{.4} \div 708.73$), if reduced in the ratio which would be required by solar expansion to Earth's mean perihelion, (207.58), would give $v_1=6.45$. Combining this with the velocity of v_0, (65,462.4), we obtain $v_2=664{,}100{,}000$, which is still nearer to the estimated velocity of light than the value deduced from solar rotation. This deduction appears, in some respects, more satisfactory than the one in the table, but I have preferred giving them both, in order to show that all rotation may probably be ultimately traceable to undulations having the velocity of light, and to exhibit, at the same time, the curious confirmation of my view, that "Venus may be regarded as an exterior satellite of the Earth, at a limit analogous to that of the solar system" (*ante.* xii, 409).

The accordance of v_2, for the principal planet in each of the asteroidally divided belts (♃, ⊕), with the planetary velocity at the principal centre of gravity in the entire system (⊙, ♃ *p*), is no less interesting in its way, than those which I have already noticed. It may be connected with the velocity of light, by conceiving a rotating æthereal sphere extending to the mean of the mean aphelion distances of Earth and Jupiter $\left(\frac{1.0339 + 5.4274}{2} = 3.2306\right)$. If such a sphere had the planetary velocity at the Sun's surface, its peripheral velocity would be 663,492,000, which differs by less than one-tenth of one per cent. from the value deduced for lunar v_2 in the foregoing paragraph; and less than half of one per cent. from the estimated velocity of light.

The equivalents of v_2 for Saturn and Venus, are interesting from their introduction of the ratio of the quantity of heat under constant pressure, to the quantity under constant volume, (1.421 : 1). An æthereal sphere extending to the linear centre of oscillation between Saturn's mean aphelion and Jupiter's mean distance, $\left(\frac{2 \times 5.203 + 10}{3} = 6.802\right)$, and rotating with the velocity of Saturn's v_2 at the Sun's surface, would have a peripheral velocity of 661,659,000, which differs by less than one-fifth of one per cent. from the estimated velocity of light. A similar sphere for Venus, limited by the mean between the same Saturnian centre of oscillation and Earth's mean distance, ($\frac{10}{3} + \frac{1}{2} = 3.83$), would have a peripheral velocity of 659,608,000, differing less than one-eighth of one per cent. from the luminous velocity.

The values of v_2 for the exterior planets of the Telluric belt, (☿, ♂), are in simple harmonic relations to the planetary velocity at the Sun ($\frac{3}{4}$ ⊙) and to Venus's v_2 ($\frac{1}{3}$ ♀). The radii, of the æthereal spheres which are determined by the peripheral velocity of light and the solar superficial velocity of v_2, are dependent on the mean aphelion reverse linear centre of oscillation of Saturn ($\frac{1}{3}$ of 10) and the mean distance of Jupiter, for Mercury, $\left(\frac{\text{c.o.}\,♄ + ♃}{2}\right)$ and Saturn's mean perihelion reverse centre of linear oscillation ($\frac{1}{3}$ of 9.078) and Uranus's mean aphelion (20.0432) for Mars,

$\left(\frac{\text{c. o. ♄} + ♅}{2}\right)$. The latter coincidence is within less than one seventieth of one per cent.

The synchronism, between light oscillations from Sun to Uranus's mean aphelion and planetary revolution at Sun's surface, lends interest to the following approximate ratios of the spheral radii to Uranus's mean radius vector: ☿, $\frac{2}{9}$; ♀, $\frac{1}{5}$; ⊕, $\frac{1}{6}$; ♂, $\frac{3}{5}$; ♃, $\frac{1}{6}$; [♄ $\frac{1}{3}$ aphelion *r. v.*]

Æthereal Spheres of Rotary Undulation.

Spheral *r*.		Periph'l *v*.	Vel. of light.
☿ (♄ *c. o.*,* ♃)	mean.	657,654,500	660,434,400.
♀ (♄ *c. o.*, ⊕)	"	659,608,000	"
⊕ (⊕, ♃)	"	663,492,000	"
♂ (♄ *c. o.*, ♅)	"	660,348,000	"
♃ (⊕, ♃)	"	663,492,000	"
♄ (♄, ♃)	c. o.	661,659,000	"

Comparison of Harmonic and Actual Spheral Radii.

	Approximate.				Actual.
☿	$\frac{2}{9}$ ♅ mean	915.91	× ☉ *r*.		920.91
♀	$\frac{1}{5}$ " "	824.32	"		824.66
⊕	$\frac{1}{6}$ " "	686.93	"		696.11
♂	$\frac{3}{5}$ " "	2472.95	"		2478.64
♃	$\frac{1}{6}$ " "	686.93	"		696.64
♄	$\frac{1}{3}$ " aphelion,	1435.50	"		1458.75

IV. WEATHER STUDY.

In the "American Weather Notes," which I had the honor of communicating to the Society at its meeting of March 3, 1871, I first called attention to the comparative frequency of anti-cyclonic storms, and to some other pecularities of meteorological phenomena which indicate the importance of regarding Espy's lines of indraught, as well as Redfield's centres of cyclonism, in making weather forecasts. I subsequently showed (*ante*, xii., 65, 123) that the general atmospheric movement in America is anti-cyclonic, while in western Europe, where I had looked for very marked cyclonism, anti-cyclonic are nearly as frequent as cyclonic currents; and that two of our principal storm centres are situated near normal intersections of polar and equatorial currents.

An abstract of my views was subsequently published in the manual of the Signal Service Bureau, and the officers of the Bureau have communicated to the public journals some remarkable evidences of anti-cyclonism in storms of great magnitude. It therefore seems desirable to ascertain the extent of this apparent exception to the generally received law of storms, and I have accordingly undertaken some special study of the weather maps, in order to ascertain how far my ideas are sustained by two

* It is well to remember that Saturn's centre of linear oscillation is at the centre of gravity of the planetary system, and its reverse centre is at the octave node of the centre of gravity.

years' additional systematic observations. The results of such study seem to me to be important enough to justify others in entering on a similar examination, and I hope the investigation may interest a sufficient number of inquirers to successfully eliminate any influence of personal bias, and to establish new and valuable meteorological laws.

My weather maps are all of the morning issue, exhibiting the daily official returns of thermometer, barometer, wind, cloud, rain, and snow, at 7.35, A. M., Washington mean time. In order to estimate the comparative frequency of cyclonic and anti-cyclonic storms, I first arranged, in respective columns, such as belonged unmistakably to either class. When it was impossible to determine the character of the current, I counted all that were at and below the barometric mean, as cyclonic; all that were near the crest of the barometric wave, as anti-cyclonic; all that were still left in doubt, as adding equal weight to each column. This classification, based on Ferrel's well known-principles, gave "the benefit of the doubt" in all instances to the cyclonic theory, and, if it is chargeable with partiality in either direction, I hope the partiality will prove to be towards the views which are in opposition to my preconceived opinions.

The number of observations is probably insufficient, and the extent of the comparison too limited to justify any very general conclusions, but there appears to be sufficient consistency in the results to render it probable, that even after storms are well developed and considerable precipitation has taken place, the currents continue anti-cyclonic in proportions varying from one-fifth to more than two-fifths of the whole number of observations, the proportion being greater in winter than in summer. The following table exhibits the number of morning rains (R) and snows (S) classed as cyclonic (C), and anticyclonic (A), together with the normal percentages of each, as determined by a simple smoothing of the resulting curves:

CLASSIFICATION OF AMERICAN STORMS.

	Observed.				Normals.			
	C R	A	C S	A	C R	A	C S	A
January	30	14	48	33	67	33	61	39
February	38	16	39	33	70	30	62	38
March	35	11	47	16	70	30	68	32
April	20	14	8	4	69	31	76	24
May	31	10	4	0	70	30	78	22
June	23	8			71	29		
July	18	10			71	29		
August	21	6			73	27		
September	23	9			73	27		
October	24	8	3	2	71	29	72	28
November	39	21	37	20	67	33	66	34
December	30	21	46	25	64	36	62	38
Winter	201	99	185	115	67	33	62	38
Spring	209	91	222	78	70	30	74	26
Summer	215	85			72	28		
Autumn	211	89	138	62	70	30	69	31
Year	836	364	545	255	70	30	68	32

In a few instances a line of rainfall with double curvature has been strikingly marked, extending almost from the summit of an atmospheric crest to the foot of an adjacent valley, thus exemplifying the accuracy of Ferrel's deductions, and encouraging the belief that close and systematic study of the tri-daily maps, under the guidance of those deductions, will render the future success of our Signal Service Bureau even more marvelous than the past.

Some stations seem to be exposed to peculiar local anti-cyclonic influences. Although the number of stations is insufficient for an entirely satisfactory study of such influences, and although I have given no special attention to their investigation, having merely noted a few of the anomalies which seemed most striking, I will venture to suggest a careful tabulation and examination of reports from Shreveport, Lynchburg, Denver, Cheyenne, Pittsburg, and St. Louis, with a view to subjecting any exceptional indications, which they may furnish, to a rigid scrutiny. I have sometimes been inclined to attribute irregularities, which seemed to be of a systematic character, to imperfections in the instruments of observation, but the careful comparisons to which they are subjected before leaving the Washington office, seem to make such a hypothesis less probable than the one which looks to local perturbations, originating in peculiarities of physical position.

The tendency to parallelism of atmospheric currents, both in vertical and horizontal planes, seems to be indicated in a large majority of the maps. The isolation of opposite horizontal currents appears, however, to be more complete than that of the vertical currents. So generally is this the case that I doubt if there is *ever* any considerable blending of upper and lower strata of air, except when a partial vacuum has been brought about by great condensation of vapor through the whole height of contiguous polar and equatorial currents, which are moving in opposite directions in the same stratum. If my belief is well founded, the cyclonism or anti-cyclonism of storms at their beginning should depend mainly on the relative position of the mixing currents. As a general rule, if the condensing current is east of the vapor-saturated current or between the vapor current and the equator, the storm should be anti-cyclonic, unless, and until, local precipitation has been copious enough to reverse the normal direction of the blending winds. If the relative position of the vapor-bearing and cooling currents is reversed, the storm should become more speedily and increasingly cyclonic, the precipitation should be more rapid, the winds more violent and tempestuous, the conditions in other respects, such as pressure, vapor-saturation; temperature, and velocity of wind at the outbreak of the storms, being the same. These views seem to be confirmed by the facts which I have adduced in previous papers, as well as by the special examination of which I am now treating, but for that very reason I prefer to submit them, by simple statement, for examination and test by others, who are either wholly free from any prepossession, or whose bias is different from my own; my wish

being rather to get at the truth, than to build up or overturn any plausible theory.

On some accounts it would be desirable to have observing stations as thickly scattered over our whole domain, as they are in England and some other European countries. But on the whole, I think, that at least for the time being, the gain is greater than the loss in having such diffusion of reports as will facilitate the early discovery of practical general laws, the influence of which might be obscured by the perplexities of local disturbances, if the stations were twice as numerous. A great part of the success of American meteorology, to which some of our foreign competitors have given the palm with almost envious reluctance, may have sprung from the broad generalizations which were at first forced upon the Meteorological Committees of the American Philosophical Society and the Franklin Institute, by the magnificent breadth of the Mississippi Valley, and the consequent unbroken sweep afforded to the winds that break over the ridges of the Rocky Mountains, or pour from the Arctic Regions through the Valley of the Saskatchewan and across the great lakes. The time will, however, soon come, if it has not already come, when the need of more minute details will become evident, and when State Legislatures, Scientific bodies, or local Boards of Trade, should devise means for supplementing, or use their influence for widely enlarging, the magnificent work of the National Bureau.

The plan of recording observations at all the stations at the same actual time, is undoubtedly the best for determining the mechanical forces which are operating for producing meteorologic changes, and consequently for the general purposes of weather forecast. It is, however, attended with some inconveniences, especially in regard to temperature, the estimated direction of isabnormal thermograms being liable to error on account of inadequate allowances for differences of local time. The importance of those thermograms may be readily understood. The barograms determine the gravitating influences of the moment, and the forces that are now operating to effect the changes of a few succeeding hours; but if we wish to estimate the probabilities for a day or more in advance, we must regard the probable changes in the barograms themselves. Those changes are mainly dependent, either directly or indirectly, upon changes of temperature, the direct influence being manifested by increase or diminution of atmospheric density; the indirect, by condensation of moisture, in the form of rain or snow, and all the other attendant phenomena. Each centre, line, or area of precipitation, becomes a pivot or link between the upper and lower aerial currents, since there must be a downthrust, as well as an indraught, to supply the place of the condensed vapor. The degree and the rapidity with which the lower will be affected by the upper currents, will depend upon the number, position, and extent of storm areas; and a knowledge of the difference of temperature from the normal temperature for the month at each station, would

be of great service in fixing the probable sites of precipitation, and the probable cyclonism or anti-cyclonism of the winds.

If the molecular, or elastic forces are so important as they seem to be, not only in the atmospheric and ocean tides, but even in determining the arrangement, masses and motions of cosmical bodies, it is desirable that they should be made the subject of special and critical study in connection with meteorologic changes.

V. THE PLANETARY NODE BETWEEN MERCURY AND VULCAN.

Any speculations upon the probable position of a planet between Mercury and Vulcan may seem premature, so long as the existence of Vulcan itself is so very problematical. I am inclined to believe that any planetoid bodies that may be found between Mercury and the Sun will prove to be very minute, perhaps of an order of magnitude like the asteroids. It is true that the harmonic coincidences on which I have based my hypothesis of an inter-Mercurial pair, may be merely accidental, but when the agreement is so curious and close as I have shown it to be, it is surely well to see whether we can find any other evidence, either to confirm or contradict its manifest indications.

Near $\frac{7}{10}$ of Jupiter's perihelion (3.485) is the first term of the terrestrial harmonic series (3.5).

Near $\frac{7}{10}$ of Earth's mean distance (.7) is Venus's mean distance (.723).

Near $\frac{7}{10}$ of Mercury's mean distance (.271) is the hypothetical planetary node (.269).

Near 3 times the hypothetical nodal distance (.807) is the octave node of Venus's and Earth's perihelia (.832).

Near 9 times the hypothetical nodal distance (2.421) is the outer node of Sun and Jupiter's perihelion (2.494).

Near 27 times the hypothetical nodal distance (7.263) is the octave node of Jupiter's perihelion and Saturn's mean distance (7.259).

Near 81 times the hypothetical nodal distance (21.789) is the spheroidal centre of gyration of Neptune's perihelion, as referred to Saturn's perihelion, (22.138).

Near 81 times the distance of Sun—Jupiter perihelion centre of gravity (81.56) is Mercury's mean distance.

The ratio of Saturn's mean aphelion to Earth's mean distance (10), is nearly a mean proportional, between the ratio of Vulcan's harmonic distance (.209) to the linear centre of oscillation of solar retardation (.169), and the ratio of Mercury's mean distance to the distance of the solar-Jovian perihelion centre of gravity ($\frac{209}{169} \times 81.56 = 10.043^2$).